1. (40 points) **Result sizes**

(a) Construct instances for tables $R(A, B)$ and $S(A, B)$ such that $R \cup S$ contains 4 tuples and $R \cap S$ contains 2 tuples. (You can use integers for attribute values.)

(b) Construct instances for tables $R(A, B)$ and $S(B, C)$ so that $R \times S$ contains 12 tuples and $R \bowtie S$ contains 6 tuples. Please show the result of $R \bowtie S$ in addition to $R$ and $S$. (You can use integers for attribute values.)

(c) Given a table $R(A, B)$ containing $n$ tuples, what is the largest possible size of $\Pi_A(R)$? What is the smallest possible size $\Pi_A(R)$? Explain the conditions under which these minimum and maximum cases occur.

(d) Given a table $R(A, B)$ containing $n$ tuples, what is the largest possible size of $\sigma_{a=5}(R)$? What is the smallest possible size $\sigma_{a=5}(R)$?

2. (20 pts) **Translating relational algebra expressions**

Consider the following schema:

- **Suppliers(sid, sname, address)**
- **Parts(pid, pname, color)**
- **Catalog(sid, pid, cost)**

The domain of each field is listed after the field name. Naturally, the Suppliers and Parts relations represent supplier entities and part entities and their attributes. The Catalog relation describes which suppliers sell which parts, and at what cost. Describe in words the output of these relational algebra expressions:

(a) $\pi_{\text{sname}}(\pi_{\text{sid}}(\pi_{\text{pname}}=\text{widget}(\text{Parts}) \bowtie \sigma_{\text{cost}=10}(\text{Catalog})) \bowtie \text{Suppliers})$

(b) $\pi_{\text{sname}}(\sigma_{\text{color}=\text{red}}(\text{Parts}) \bowtie \sigma_{\text{cost}<100}(\text{Catalog}) \bowtie \text{Suppliers}) \cup \pi_{\text{sname}}(\sigma_{\text{color}=\text{green}}(\text{Parts}) \bowtie \sigma_{\text{cost}<100}(\text{Catalog}) \bowtie \text{Suppliers})$
3. (10 pts) **Relational algebra to SQL**

Given relational tables with schemas $R(A, B), S(A, C), T(C, D)$, express the following relational algebra query in SQL. That is, write an SQL query that returns exactly the same result as the algebra query below:

$$\Pi_{R.A,R.B,S.C}(\sigma_{R.A<10,T.D=11}(R \bowtie S \bowtie T))$$

(Note: There is a slightly tricky part to this question—remember that the relational algebra and SQL handle duplicates differently.)

4. (30 pts) **Beginning SQL exercises**

The following schema represents a subset of information from a social networking site. The Friend table lists all the friends of some particular user. The Pages table lists pages of various types. The Likes table records the pages that friends like.

- **Friend(fid, first_name, last_name, birthday, hometown, gender, relationship_status)**
- **Likes(fid, pid, date_liked)**
- **Page(pid, name, category)**

Write SQL expressions for each of the following queries. A sample database instance is available for you to run your queries. You can open the database in SQLite with the following command:

```bash
elnux1> sqlite3 /courses/cs400/cs445/cs445.f2010/sample/simple_facebook.db
```

Don’t forget to terminate SQL statements with a semicolon. Run `.help` at the prompt for instructions on SQLite. **Please submit for each query below (i) a valid SQL query, and (ii) the output of the query on the sample data.**

1. List the first and last names of all married friends.
2. Find the name and category of each page that is liked by some friend.
3. Find the first and last names of all friends who like the movie 'Inception'.

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