# High-level overview: Indexes

<table>
<thead>
<tr>
<th>id</th>
<th>age</th>
<th>salary</th>
<th>other</th>
</tr>
</thead>
<tbody>
<tr>
<td>006</td>
<td>19</td>
<td>50k</td>
<td>...</td>
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<tr>
<td>005</td>
<td>20</td>
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<td>004</td>
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<td>007</td>
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<td>80k</td>
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<td>70k</td>
<td>...</td>
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<tr>
<td>001</td>
<td>40</td>
<td>65k</td>
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</tr>
</tbody>
</table>

- **data file = index file**
- **clustered index**

- **index file**
- **unclustered index**
Clustered Index

- File is sorted on the index attribute
- Only one per table
Unclustered Index

- Several per table

Index File

<table>
<thead>
<tr>
<th>10</th>
<th>10</th>
<th>20</th>
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<tbody>
<tr>
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Data File

<table>
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<td>30</td>
</tr>
</tbody>
</table>
Hash-Based Index

Good for point queries but not range queries

h2(age) = 00

h2(age) = 01

h1(sid) = 00

h1(sid) = 11
B+ Trees

- Search trees

- Idea in B Trees
  - Make 1 node = 1 block
  - Keep tree balanced in height

- Idea in B+ Trees
  - Make leaves into a linked list: facilitates range queries
Leaf pages contain *data entries*:

- Data entries are *sorted* by the search key value
- Leaf pages are chained using prev & next pointers
B+ Tree Indexes

Non-leaf pages have *index entries*, used **only** to direct searches.

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B+ Tree: Most widely used Index

- Height-balanced with arbitrary inserts/deletes

- Fanout (F): number of child pointers of non-leaf node

- Height: $H = \log_F N$
  - $N =$ number of leaves
B+ Tree: Most widely used Index

- Minimum 50% occupancy (except for root)

- Each node contains m entries
  - Can be computed using page size, key size, pointer size.

Index Entries
(Direct search)

Data Entries
("Sequence set")
B+ Tree: Most widely used Index

- Parameter \( d = \text{the order} \)
- Each interior node has \( d \leq m \leq 2d \) keys (except root)

Each node also has \( m+1 \) pointers

- Each leaf node has \( d \leq m \leq 2d \) keys

Next leaf

Data records
Partitioning of an ordered domain

The search structure (non-leaf part) forms a partitioning of an ordered domain (e.g., integer, string)
Searches in a B+ Tree

Root

Entries < 17

Entries ≥ 17

Data entries are sorted.
Searches in a B+ Tree

find key 24

24 > 17
24 < 27
Insertions

- Find correct leaf $L$ via a top-down search.
- Put data entry onto $L$.
  - If $L$ has enough space, *done*!
  - Else, must **split** $L$ (*into $L$ and a new node $L_2$*)
    - Redistribute entries evenly, **copy up** middle key $k$,
      insert $(k, \text{pointer to } L_2)$ into parent of $L$.
  - Splitting can happen recursively to **non-leaf nodes**
    - Redistribute entries evenly, but **push up** middle key.
      (Contrast with leaf splits.)
- Splits “grow” the tree!
  - First **wider**, then **one level taller** when the root splits.
Example

Inserting 8*
Example

Inserting 8*

Entry to be inserted in parent node. (Note that 5 is copied up and continues to appear in the leaf)
Entry to be inserted in parent node. (Note that 17 is pushed up and only appears once in the index.)
After Insertion

Root has split leading to increased height
Deletions

- Start at root, find leaf $L$ where entry belongs.
- Remove the entry.
  - If $L$ is at least half-full, done!
  - If $L$ has only $\lfloor \frac{n}{2} \rfloor - 1$ entries,
    - Try to re-distribute, borrowing from sibling (adjacent node with same parent as $L$).
    - If re-distribution fails, merge $L$ and sibling. Must delete index entry (pointing to $L$ or sibling) from parent of $L$.
- Merge could propagate to root, decreasing height.
Example

Delete 19*
Delete 20*
After Deletions

Root

2* 3* 5* 7* 13* 8* 14* 16* 22* 24* 27* 29* 33* 34* 38* 39*

17

27 30

5 13
Example

Delete 24*
... deleting 24*

- Must merge nodes.
- **Toss** index entry (right)
- **Pull down** of index entry (below).
Example of Non-leaf Re-distribution

Tree is shown above during deletion of 24*. (What could be a possible initial tree?)

In contrast to previous example, can re-distribute entry from left child of root to right child.
Intuitively, entries are re-distributed by ‘pushing through’ the splitting entry in the parent node.

It suffices to re-distribute index entry with key 20; we’ve re-distributed 17 as well for illustration.
Prefix Key Compression

- Important to increase fan-out. (Why?)
- Key values in index entries only `direct traffic`; can often compress them.
  - E.g., adjacent index entries with search key values
    
    [Dave Jones, David Smith, Devarakonda Murthy]
  - Can we abbreviate David Smith to Dav?
    - Not correct! Can only compress David Smith to Davi.
    - In general, while compressing, must leave each index entry greater than every key value (in any subtree) to its left.
- Insert/delete must be suitably modified.
Bulk Loading of a B+ Tree

- Already have a large collection of records. Want to create a B+ tree. Doing so by repeatedly inserting records?
  - Slow due to repeated traversals and splits.
  - Not necessarily the optimal structure. An example?
  - Low storage utility. An example?

- **Bulk Loading** can be done much more efficiently!
Bulk Loading Algorithm

- **Initialization:**
  - Sort all data entries
  - Insert pointer to the first (leaf) page in a new (root) page.

```
3* 4* 6* 9* 10* 11* 12* 13* 20* 22* 23* 31* 35* 36* 38* 41* 44*
```

Sorted pages of data entries; not yet in B+ tree
Bulk Loading Algorithm (Contd.)

- Index entries for leaf pages always go into r*, right-most index page just above the leaf level.
- When the r* node fills up, it splits.
- Split may go up the right-most path to the root.
Multiple Inserts vs. Bulk Loading

- **Multiple inserts:**
  - Slow due to I/O cost (and locking) overhead.
  - No sequential storage of leaf pages.
  - Sometimes low storage utility.

- **Bulk Loading:**
  - Fewer I/Os during build.
  - Leaf pages will be stored sequentially (and linked).
  - Can control “fill factor” on pages.
The Database Tuning Problem

- We are given a workload description
  - List of queries and their frequencies
  - List of updates and their frequencies
  - Performance goals for each type of query
- Perform physical database design
  - Choice of indexes
  - Tuning the conceptual schema
    - Denormalization, vertical and horizontal partition
  - Query and transaction tuning
The Index Selection Problem

- Given a database schema (tables, attributes)
- Given a “query workload”:
  - Workload = a set of (query, frequency) pairs
  - The queries may be both SELECT and updates
  - Frequency = either a count, or a percentage

- Select a set of indexes that optimizes the workload

  In general this is a very hard problem
Index selection decisions

- To index or not to index?
- Which key?
- Multiple keys?
- Clustered or unclustered?
- Hash or trees?
Index Selection: Which Search Key

- Make some attribute K a search key if the WHERE clause contains:
  - An exact match on K
  - A range predicate on K
  - A join on K
The Index Selection Problem 1

V(M, N, P)

Your workload is this

100,000 queries:

```
SELECT * 
FROM V  
WHERE N=?
```

100 queries:

```
SELECT *  
FROM V   
WHERE P=?
```

What indexes?
The Index Selection Problem 1

V(M, N, P)

Your workload is this

100,000 queries:

100 queries:

SELECT * FROM V WHERE N=?

SELECT * FROM V WHERE P=?

A: V(N) and V(P) (hash tables or B-trees)
The Index Selection Problem 2

V(M, N, P)

Your workload is this

100,000 queries:

```
SELECT * 
FROM V 
WHERE N>? and N<?
```

100 queries:

```
SELECT * 
FROM V 
WHERE P=?
```

100,000 queries:

```
INSERT INTO V 
VALUES (?, ?, ?)
```

What indexes?
The Index Selection Problem 2

V(M, N, P)

Your workload is this

100,000 queries:

```
SELECT * FROM V WHERE N>? and N<?
```

100 queries:

```
SELECT * FROM V WHERE P=?
```

100,000 queries:

```
INSERT INTO V VALUES (?, ?, ?)
```

A: definitely V(N) must B-tree; unsure about V(P)
The Index Selection Problem 3

\[ V(M, N, P) \]

Your workload is this

100,000 queries:

\[
\text{SELECT } * \text{ FROM } V \text{ WHERE } N = ?
\]

1,000,000 queries:

\[
\text{SELECT } * \text{ FROM } V \text{ WHERE } N = ? \text{ and } P > ?
\]

100,000 queries:

\[
\text{INSERT INTO } V \text{ VALUES (?, ?, ?)}
\]

What indexes?
The Index Selection Problem 3

\[ V(M, N, P) \]

Your workload is this

100,000 queries:

SELECT * FROM V WHERE N=?

1,000,000 queries:

SELECT * FROM V WHERE N=? and P>?

100,000 queries:

INSERT INTO V VALUES (?, ?, ?)

A: \[ V(N, P) \]
The Index Selection Problem 2

V(M, N, P)

Your workload is this

1,000 queries:

```
SELECT * 
FROM V 
WHERE N>? AND N<?
```

100,000 queries:

```
SELECT * 
FROM V 
WHERE P>? AND P<?
```
The Index Selection Problem 2

$V(M, N, P)$

Your workload is this:

1,000 queries:

```sql
SELECT * 
FROM V
WHERE N>?
and N<?
```

100,000 queries:

```sql
SELECT * 
FROM V
WHERE P>?
and P<?
```

A: $V(N)$ unclustered; $V(P)$ clustered
Basic Index Selection Guidelines

- Consider queries in workload in order of importance

- Consider relations accessed by query
  - No point indexing other relations

- Look at WHERE clause for possible search key

- Try to choose indexes that speed-up multiple queries

- And then consider the following...
Index Selection: Multi-attribute Keys

- Consider creating a multi-attribute key on \( K1, K2 \), if ...
  - WHERE clause has matches on \( K1, K2 \), ...
    - But also consider separate indexes
  - SELECT clause contains only \( K1, K2 \), ..
    - A covering index is one that can be used exclusively to answer a query, e.g. index \( R(K1,K2) \) covers the query:
      - Can be answered with an index-only plan
        ```
        SELECT K2
        FROM R
        WHERE K1=55
        ```
To Cluster or Not to Cluster?

- Range queries benefit mostly from clustering

- Covering indexes do not need to be clustered

Why?
Percentage of tuples retrieved vs Cost relationship:

- **Sequential scan**
- **Unclustered index**
- **Clustered index**

SQL Query:

```sql
SELECT * 
FROM R 
WHERE K>? and K<? 
```
Updates

- Indexes speed up queries
  - SELECT FROM WHERE

- But they usually slow down updates:
  - INSERT, DELETE, UPDATE

- However some updates benefit from indexes

```
UPDATE R
SET A = 7
WHERE K = 55
```
Hash Indexes
Hash Table v.s. B+ tree

- **Rule 1:** always use a B+ tree 😊

- **Rule 2:** use a Hash table on K when:
  - There is a very important selection query on equality (WHERE $K=?$), and no range queries
  - You know that the optimizer uses a nested loop join where $K$ is the join attribute of the inner relation (you will understand that in a few lectures)
Hash Indexes

- **Hash-based** indexes are best for *equality selections*. **Cannot** support range searches.
  - E.g., retrieve a student with id ‘123’ or all students at age=20.

- Static and dynamic hashing techniques exist.

- As for any index, 3 alternatives for data entries $k*$:
  - $<k, \text{data record with key value } k>$
  - $<k, \text{rid of data record with search key value } k>$
  - $<k, \text{list of rids of data records with search key } k>$
Static Hashing

- $h(k) \mod N = \text{bucket to which data entry with key } k \text{ belongs. } k_1 \neq k_2 \text{ can lead to the same bucket.}$

- **Static structure**: # buckets (N) fixed
  - *Primary pages*: allocated sequentially, never de-allocated;
  - *Overflow pages*: allocated/de-allocated if needed.
Static Hashing (Contd.)

- Hash function on the search key distributes values over [0 ... N-1].
  - \( h(key) \mod N = (a \ast key + b) \mod N \)
  - \( a \) and \( b \) are constants; a lot is known about how to tune \( h \)

- Buckets contain data entries in a chain of pages.
  - Long overflow chains degrade performance.
  - Dynamic techniques fix this problem.
Extendible Hashing

- When bucket (primary page) becomes full, why not re-organize file by *doubling* num. of buckets?
  - Reading and writing all pages is expensive!

- **Idea**: use a *directory of buckets*. When bucket is full:
  1) *double the directory*,
  2) *split just the bucket that overflowed*.

- Directory much smaller than file, so doubling is cheap.
- Only one page of data entries is split. *No overflow page!*
- Trick lies in how hash function is adjusted.
Example

- Directory is array of size $N=4$, *global depth* $D = 2$.

- To find bucket for key:
  1) get $h(key)$,
  2) take last *global depth* # bits of $h(key)$, i.e., mod $2^D$.
  - If $h(key) = 5 =$ binary $101$,
  - Take last 2 bits, go to bucket pointed to by 01.

- Each bucket has *local depth* $L$ ($L \leq D$) for splits!
Inserts

- If bucket is full, **split** it:
  - Allocate new page,
  - Re-distribute,
  - If needed, **double** directory.

- Double the directory if **global depth** $D = \text{local depth} \ L$
  - Split if $D = L$.
  - Otherwise, don’t.

*Insert $k^*$ with $h(k) = 20?*
Insert $h(k) = 20$ (Causes Doubling)

**LOCAL DEPTH**

**GLOBAL DEPTH**

**DIRECTORY**

Bucket A ($000$)

Bucket B

Bucket C

Bucket D

Bucket A2 ($100$)

(`split image` of Bucket A)

**LOCAL DEPTH**

**GLOBAL DEPTH**

**DIRECTORY**

Bucket A

Bucket B

Bucket C

Bucket D

Bucket A2

(`split image` of Bucket A)
Points to Note

- 20 = binary 10100. Last 3 bits needed to distinguish A, A2.
  - *Global depth D of directory*: Max # of bits needed to tell which bucket an entry belongs to.
  - *Local depth L of a bucket*: Actual # of bits needed to determine if an entry belongs to this bucket.

- Bucket split causes directory doubling if before insertion, $L$ of bucket = $D$ of directory.
Deletes

- Remove a data entry from bucket
  - If bucket is empty, can be merged with `split image`.
  - If each directory entry points to same bucket as its split image, can halve directory.
  - If assume more inserts than deletes, do nothing...
Comments on Extendible Hashing

- **Access cost**: If directory fits in memory, equality search takes one I/O to access the bucket; else two.

- **Skews**: If the distribution of hash values is skewed, directory can grow large. An example?

- **Duplicates**: Entries with *same key value* need overflow pages!
Linear Hashing

- Dynamic hashing, an alternative to Extendible Hashing.
- Advantage: adjusts to inserts/deletes without a directory.

**Idea**: Use a family of hash functions $h_0, h_1, h_2, \ldots$
- $h_i(key) = h(key) \mod (2^i * N)$; $N =$ initial # buckets
- $h$ is some hash function (range is *not* 0 to N-1)
- $h_0 = h(key) \mod N$
- $h_{i+1}$ doubles the range of $h_i$ (similar to directory doubling)
- If $N = 2^{d0}$, for some $d0$, $h_i$ consists of applying $h$ and looking at the last $di = d0 + i$ bits.
Linear Hashing (Contd.)

- LH avoids directory by
  1) using *temporary* overflow pages, and
  2) choosing bucket to split in a *round-robin* fashion.

- Splitting proceeds in `rounds’ . Round $L$, $N_L = 2^L N$ buckets:
  - *Next* bucket to be split: Buckets $[0, Next-1]$ have been split; $[Next, N_L]$ yet to be split.
  - Round $L$ ends when all $N_L = 2^L N$ initial buckets have been split.
L_{th} \text{ Round of splitting}

Bucket to be split \textit{Next}

$2^L N$ buckets at the beginning of this round. This is the range of $h_L$.

`Split image' buckets: created (through splitting buckets) in this round.

Buckets split in this round
Searches

Bucket to be split \textbf{Next}

$2^L N$ buckets at the beginning of this round. This is the range of $h_L$.

\textbf{Search:} To find bucket for data entry $k^*$, apply $h_L(k)$:

- If $h_L(k)$ in range $[\text{Next}, N_L]$, $k^*$ belongs here.
- Else, apply $h_{L+1}(r)$ to choose between bucket $h_L(k)$ and its split image bucket $h_L(k) + N_L$.
Inserts

- **Insert**: Find bucket $B$ by applying $h_L / h_{L+1}$. If $B$ is full:
  - Add overflow page, insert data entry $k^*$.
  - *(Maybe)* split bucket $\text{Next}$ (often $B \neq \text{Next}$) using $h_{L+1}$, increment $\text{Next}$.
  - Can choose any criterion to `trigger` split.
- Since buckets are split round-robin, long overflow chains don’t develop!
- Compared to *Extendible Hashing*:
  - Switching of hash functions is *implicit* in how the # of bits examined is increased.
  - **No need to physically double the directory!**
Example of Linear Hashing

Round $L=0$, $N_L=4$, $h_0: \mod 2^2$

On split, $h_{L+1}$ is used to re-distribute entries.

Round $L=0$, $N_L=4$, $h_0: \mod 2^2 / h_1: \mod 2^3$
### Example: End of a Round

#### Round \( L=0, N_L=4, h_0: \text{mod } 2^2/ h_1: \text{mod } 2^3 \)

<table>
<thead>
<tr>
<th>( h_1 )</th>
<th>( h_0 )</th>
<th>PRIMARY PAGES</th>
<th>OVERFLOW PAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>00</td>
<td>32*</td>
<td></td>
</tr>
<tr>
<td>001</td>
<td>01</td>
<td>9* 25*</td>
<td></td>
</tr>
<tr>
<td>010</td>
<td>10</td>
<td>66<em>18</em>10<em>34</em></td>
<td></td>
</tr>
<tr>
<td>011</td>
<td>11</td>
<td>31<em>35</em>7<em>11</em></td>
<td>43*</td>
</tr>
<tr>
<td>100</td>
<td>00</td>
<td>44<em>36</em></td>
<td></td>
</tr>
<tr>
<td>101</td>
<td>01</td>
<td>5* 37<em>29</em></td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>10</td>
<td>14<em>30</em>22*</td>
<td></td>
</tr>
</tbody>
</table>

- **Next=3**

#### Round \( L=1, N_L=8, h_1: \text{mod } 2^3 \)

<table>
<thead>
<tr>
<th>( h_1 )</th>
<th>( h_0 )</th>
<th>PRIMARY PAGES</th>
<th>OVERFLOW PAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>00</td>
<td>32*</td>
<td></td>
</tr>
<tr>
<td>001</td>
<td>01</td>
<td>9* 25*</td>
<td></td>
</tr>
<tr>
<td>010</td>
<td>10</td>
<td>66* 18<em>10</em>34*</td>
<td></td>
</tr>
<tr>
<td>011</td>
<td>11</td>
<td>43* 35* 11*</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>00</td>
<td>44<em>36</em></td>
<td></td>
</tr>
<tr>
<td>101</td>
<td>11</td>
<td>5* 37<em>29</em></td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>10</td>
<td>14<em>30</em>22*</td>
<td></td>
</tr>
<tr>
<td>111</td>
<td>11</td>
<td>31* 7*</td>
<td></td>
</tr>
</tbody>
</table>

- **Insert 50**
- **Next=0**

---

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Summary of Dynamic Hashing

- Linear Hashing (LH) avoids directory by splitting buckets round-robin, and using overflow pages.
  - Overflow pages not likely to be long.
  - Duplicates handled easily.
- Space utilization can be lower than Extendible Hashing (EH), since splits not concentrated on `dense’ data areas.

- *Skewed* data distributions: *hash values* of data entries are not uniformly distributed! Cause problems with EH and LH.
Summary of Hashing

- **Hash-based indexes**: best for equality searches, cannot support range searches.
- **Static Hashing** can lead to long overflow chains.
- **Extendible Hashing** avoids overflow pages by splitting a full bucket when a new data entry is to be added to it. *(But duplicates may require overflow pages.)*
  - Directory to keep track of buckets, doubles periodically.
  - Can get large with skewed data; additional I/O if this does not fit in main memory.
Summary (Contd.)

- **Linear Hashing** avoids directory by splitting buckets round-robin, and using overflow pages.
  - Overflow pages not likely to be long.
  - Duplicates handled easily.
  - Space utilization could be lower than Extendible Hashing, since splits not concentrated on ‘dense’ data areas.
    - Especially true with skewed data.
    - Can tune criterion for triggering splits to trade-off slightly longer chains for better space utilization.

- For hash indexes, a *skewed* data distribution means *hash values* of data entries are not uniformly distributed! Cause problems with EH and LH.
Spatial Data
Types of Spatial Data

- **Point Data**
  - Points in a multidimensional space
  - E.g., satellite imagery, feature vectors

- **Region Data**
  - Object have spatial extend with location and boundary
  - DB typically uses geometric approximations
Spatial Queries

- **Range queries**
  - E.g., find all cities within 50 miles from UMass
  - Query has associated region
  - Answer includes overlapping or contained regions

- **Nearest neighbor queries**
  - Find the 10 cities closest to UMass
  - Results order by proximity

- **Spatial joins**
  - Find all cities near a lake
  - Expensive. Join condition involves regions and proximity
Applications

- Geographic Information Systems (GIS)
  - Geospatial information
  - All classes of queries are common
- Computer-aided design / manufacturing
  - Spatial objects (e.g., plane fuselage)
  - Range queries and spatial joins
- Multimedia databases
  - High dimensional objects in feature vector form
  - Nearest neighbor queries
Single-dimensional Indexes

- B+ trees

- When we use composite keys, we effectively linearize a higher dimensional space
Multi-dimensional Indexes

- Cluster entries to exploit near-ness in multi-dimensional space
- Keeping track of entries and maintaining a balanced structure is a challenge

Spatial clusters
Example queries

- Find hotels in a 5 mile radius of the conference venue
- Find all cities that lie on the Nile
- Given a face, find the five most similar faces
- Multi-dimensional range queries
  - $50 < \text{age} < 55$ and $80k < \text{salary} < 90k$
Difficulty

- We need an index
  - One-dimensional indexes do not support multidimensional search efficiency
  - Hash indexes only support point queries
  - Graceful inserts and deletes
The R-Tree

- An adaptation of the B+ tree

- Each key stored in a leaf is intuitively a box, or collection of intervals (one per dimension)
R-tree properties

- Leaf entry = \(<n\text{-dimensional box, rid}>\)
  - Alternative 2, with the box as the key value
  - Box is the tightest bounding box for a data object
- Non-leaf entry = \(<n\text{-dim. box, ptr to child}>\)
  - Box covers all boxes in subtree
- All leaves at equal distance from the root
- Nodes 50% full (except root)
  - Or can pick any parameter m
Example
Example
Search for overlapping box

- Start at root
  - If node is non-leaf, for each entry \(<E,\text{ptr}>\), if box \(E\) overlaps \(Q\) search subtree at \(\text{ptr}\)
  - If current node is leaf for each entry \(<E,\text{ptr}>\), if box \(E\) overlaps \(Q\), rid has an object that may overlap

May have to search several subtrees
Improving search using constraints

- Boxes can be represented compactly

- Convex polygons would be more accurate
  - Less overlap between nodes. May have to fetch fewer nodes
  - Cost of overlap test is higher
Insertions

- Start at root
- Go to best-fit leaf L
  - Go to child whose box needs the least enlargement
  - Resolves ties by picking the smallest area child
- If L has space, insert entry and stop, otherwise split L
  - Propagate splits accordingly
Splitting a node

- Entries must be distributed between L1 and L2
- Reduce likelihood that both L1 and L2 will be searched in subsequent queries
- Redistribute to minimize their total area

**bad**

**good**
R-tree variants

- $R^*$
  - Forced inserts to reduce overlap
  - Re-insert some portion of the entries

- $R^+$
  - Insert into multiple leaves
  - Single path search, but redundancy
GiST

- Abstracts the “tree” nature of a class of indexes including B+ and R-tree variants
  - The similarities between insert/delete/search make it possible to provide templates
  - B+ trees are very important in a DBMS, so they are implemented separately
  - GiST provides an alternative for implementing other tree indexes
Next lecture

- Theory
  - Advanced datalog
  - Equivalence
  - Containment